## Applications of Exponential Functions

You must complete BOTH of the following projects.

## Caution:

1. This is to be an individual assignment. You may discuss these problems with your classmates, but the work you hand in must be your own. Any assignments deemed to be copies of each other will be all assigned a mark of zero.
2. As with any assignment involving research, it may be tempting to take a short cut and not properly reference your sources, or worse, copy, paste and pass off someone else's work as your own. It is expected that any information you gather during this assignment will be properly quoted and sourced using APA format. Included with this assignment is a guide from the University of Ottawa explaining how to avoid plagiarism, and an APA citation style guide from Long Island University. Any work found to be plagiarized will also receive a mark of zero.

## This assignment is due Thursday, November $20^{\text {th }}$.

## Project One: Monod's Nightmare

(source: Mathematics Teacher, Vol.100, No.9, May 2007)
French biologist and Nobel Laureate Jacques Monod (1910-1976) is famous for his work involving bacteria, specifically Escherichia coli (E.coli). In particular, he wanted to know how much time it would take to fill a room with E.coli. E. coli are a large and diverse group of rod-shaped bacteria found in the digestive tract of warm-blooded animals (including humans). Many are completely harmless, while others can cause illness such as diarrhea, pneumonia or urinary tract infections. Transmission of illness-causing e.coli is typically caused by improper hygiene - for example, not washing one's hands after using the washroom. You may recall in 2006, there was a large outbreak of e.coli contamination in pre-packaged spinach, resulting from irrigated water contaminated with cattle feces on the producer's farm. This outbreak caused many illnesses and is suspected to have been responsible for a few deaths.

1. Suppose you have a classroom of dimensions 6 m (length), 4.3 m (width) and 2.4 m (height). How many E.coli bacteria would it take to fill the classroom? For the purpose of this activity, assume that the bacteria can be stacked one atop another and one next to another. In other words, consider each bacteria as a rectangular prism with dimensions $2,0.75$ and 0.75 microns $\left(2 \times 10^{-6} \mathrm{~m}, 0.75 \times 10^{-6} \mathrm{~m}, 0.75 \times 10^{-6}\right.$ m respectively).
2. E.coli doubles every 20 minutes.
a. Let $P$ be the number of E.coli bacteria initially present, $r$ be the rate of growth, and $t$ be the time (in hours). Find $r$ using the exponential growth model $A=P e^{\prime t}$. (hint: for A, how many will be left after $1 / 3$ of an hour?)
b. Placing $t$ on the $x$-axis, sketch a graph (well labeled, and neat!) of the function demonstrating the growth of E.coli. A will be your y-axis.
c. Using this information, calculate the time it would take to fill the room in question 1 with E.coli bacteria, assuming the population starts with just one bacterium. Why might this project have been called "Monod's Nightmare"?
d. What factors not taken into account by the exponential growth model might make it a less-than-accurate model for the growth of E.coli?

Bonus: Using the exponential growth model, how long would it take for the E.coli bacteria to cover the surface of the earth 30 cm deep?

Project Two: Pandemic Panic!
Part One: Backgrounder


In addition to the questions below, an article "Avian Flu: Should You Fear the Chicken?" has been included as background information. It may even help you to answer some of the questions.

1. In your own words, citing a source if necessary, what is a pandemic?
2. How does a pandemic differ from an epidemic?
3. Who is generally most at risk for becoming ill? Why?
4. What are the conditions for declaring a "pandemic"?
5. What societal factors may increase the spread of pandemic influenza? Describe three.
6. What measures could possibly be used to control a pandemic? Describe three.
7. How might a pandemic influence your life?

Since the beginning of the $20^{\text {th }}$ century, there have been three pandemics. In 1918, influenza subtype H1N1, also known as the Spanish Influenza, caused an estimated 50 to 100 million deaths worldwide.
5. Briefly describe the other two pandemics in the 20th century. (Hint: 1957 and 1968)

## Part Two: The Math of Pandemics

The spread of a virus is influenced by four factors:

- the size of the population of opportunity
- the number of days contagious
- the number of people with whom an infected person comes into contact; and,
- the probability of contacting the virus from contact with an infected person.

You will need to complete this section very, very carefully. Graph paper is highly suggested. Careful reading of the instructions will help as well. This will not be easy, but you can do it.

## Graph One

Here are the parameters:
Population is 200 persons.
You will begin with two contagious persons.
Once contagious, a person will remain contagious for 5 days. During this time, each person they come into contact with has a $10 \%$ chance of becoming infected. Once they have been infected, and have passed through the 5 day contagious phase, they are not able to become re-infected.

Each day, each contagious person will come into contact with 5 people. Therefore, one out of ten will become ill.

Your x-axis will be measured in days.
Your y-axis will be measured in persons.
You may not have a fraction of a person.
You will have to keep track of several things at once. Here is a table to help you.

| Day | \# contagious | \# no longer contagious | Persons remaining |
| :--- | :--- | :--- | :--- |
| 1 | 2 | 0 | 198 |
| 2 | 3 | 0 | 197 |
| 3 | 4 | 0 | 196 |
| 4 | 6 | 0 | 194 |
| 5 | 9 | 0 | 191 |
| 6 | 12 | 2 | 186 |

Note that on the sixth day, the original infected persons are no longer contagious, therefore, they are now in the second column. This means you need to keep track of who becomes no longer contagious and when! wow, that's tough!!

On your first graph, you will have two different curves - showing the number contagious, and the no longer contagious persons.

## Questions about graph one:

1. What assumptions are you making?
2. What could affect the number of persons infected that we did not take into account, and how might that change your graph?

## Hypothetical modeling questions - provide written rationale - graphs not necessary <br> Persevere! These questions are tough, but not impossible!

1. How could you change the parameters to cause the infection to die out quickly with a low percentage infected? Be specific.
2. How could you change the parameters to cause the infection to spread very quickly over time? Be specific.
3. How could you change the parameters so that about $80 \%$ of the population (160 people) is eventually infected? Be specific.
4. How could you change the parameters so that about $20 \%$ of the population is eventually infected? Be specific.

Evaluation of this assignment will use the same rubric as the Trigonometric Identities Major Assignment, however the spread of possible points will be 10 per question, rather than 2, and total presentation (for both projects) will be worth 10 as well. This is for a total of 30 marks.


